



TITLE:

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CITATION:

Mun, Se-il ...[et al]. Role of Voluntary Associations in the Improvement of Urban Central Shopping Areas. 京都大学大学院経済学研究科Working Paper 2010, 118

ISSUE DATE:

2010-06

URL:

<http://hdl.handle.net/2433/123443>

RIGHT:

# Role of Voluntary Associations in the Improvement of Urban Central Shopping Areas

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June 2010

# Role of Voluntary Associations in the Improvement of Urban Central Shopping Areas

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## Abstract

Many cities around the world suffer from the decline of central shopping areas. Voluntary associations are considered to be one of solutions to this problem. In Japan, shopping district associations have been organized in many shopping areas. Business improvement districts (BIDs) are similar organizations in North America and Europe. This paper discusses the role of voluntary associations in the revitalization of central shopping areas. We take the existence of shutdown shops into account: if there are many shutdown shops in the central shopping area, fewer consumers will frequent the open shops, because shutdown shops seem to downgrade the area's ambience. The establishment of voluntary associations increases the profit of shops in the central shopping area and the consumer surplus. The equilibrium when there is an association is, however, still inefficient, because it does not take into account the increase in consumer surplus. We also examine second-best policies to improve social welfare: optimal membership fees, matching subsidy to the association, and transport-cost subsidy to consumers. We evaluate these policies through numerical analysis.

*Keywords:* central shopping area, voluntary associations, public good, private government.

*JEL Classification Number:* R38, R51.

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# 1 Introduction

The decline of traditional central shopping areas is a serious problem in many cities around the world. Suburbanization, which was enabled by motorization, set the stage for the development of suburban shopping malls, which consumers can easily drive to and park at, and from which they can purchase everything they need. Consequently, the central shopping areas in many cities have lost customers, and many shops within them went out of business. The closure of shops has made the areas less attractive for customers, which further accelerated the decline.

Governments have recognized the problem and have made serious efforts to revitalize these areas by demolishing old buildings, improving streetscapes, adding street lights, and other methods. However, these attempts have not always been successful. Besides, many local governments are in financial straits, and thus do not have enough resources to do the work necessary to revitalize these areas. Voluntary associations formed by firms, landowners, or households are often suggested as a solution to this problem.

In Japan, shopping district associations were organized in many shopping areas. Shops located along one street formed a shopping district association that would provide services such as shopping arcades, new pavement, security, and event coordination. In North America, landowners and businesses in the city center formed business improvement districts (BIDs). BIDs resemble the Japanese shopping district associations. They provide security and sanitation services and improve the district's environment. Furthermore countries in Europe and around the world have recently introduced BIDs. In this study, we analyze the role of these organizations in urban (re)development.

Helsley and Strange [5] call voluntary organizations that provide collective goods "private governments." They define these organizations as (1) voluntary, (2) exclusive, and (3) self-financing organizations that (4) supplement services provided by the public sector and (5) behave strategically. In their model, a private government is a group of firms or households that are not satisfied with the services provided by the public sector. They show that the public sector reduces the provision of public good in response to the existences of a private government. This is because public goods provided by a private government and those provided by the public sector are substituted. The equilibrium with a private government is not necessarily Pareto superior to the equilibrium without it; however, they conclude that the effects of private government on social welfare are therefore ambiguous.

Billings [2] has presented a theoretical study of BIDs. He focuses on two

factors that influence BIDs' actions: government intervention in the formation of BIDs and competition with the suburban mall. Government may control the size of a BID when it is formed. Billings claims that government control actually makes the level of public goods provided by a BID inefficient. He also analyzes the competition between the suburban shopping mall and the traditional shopping area, where a BID can be created. The mall and the shopping area play a two-stage game. In the first stage, the shopping area decides whether to establish a BID. The mall chooses whether to enter or not at the second stage. Billings thus finds three kinds of equilibria: one with no BID and no mall, one with a BID and no mall, and one with a BID and a mall. He explains the conditions necessary for each equilibrium to emerge. Billings does not, however, investigate the effects of BID formation on economic welfare or the evaluation of alternative forms of government intervention.

This paper discusses the role of the shopping district association in the revitalization of central shopping areas. We show that the formation of a shopping district association improves the welfare of shops in the area and of consumers, compared with areas in which there are no associations, but the outcome is still inefficient. Thus we examine alternative forms of policy intervention that may be used to attain outcomes that are more efficient.

Our model is different from earlier works in two ways. First, we take into account the existence of shutdown shops in the central shopping area. When many shops in an area are shut down, the area becomes less attractive to consumers because the variety of goods they can purchase is smaller. The existence of shutdown shops also downgrade the ambience of the area. This is a negative externality to the open shops in the area. Second, we conduct welfare analysis of shopping district associations. We evaluate three alternative policies that may help these associations, prevent the decline of the central shopping area, and improve social welfare.

We present this model in Section 2, and compare the equilibrium when there is an association (we call it the equilibrium with association hereafter) and the equilibrium when there is no association (the equilibrium without association hereafter) in Section 3. We compare the equilibrium and the first-best allocation in section 3.3. Section 4 examines three second-best policies: optimal tax from shops, matching subsidy to the association, and transport-cost subsidy to consumers. We evaluate these policies with a numerical example in Section 5. Our conclusions are presented in Section 6.

## 2 Model

Suppose one-dimensional geographic space. Consumers are located according to a given density function,  $n(l)$  ( $n(l) > 0$ ), where  $l$  represents the location,  $l \in [0, L]$ . The central shopping area (Area  $C$  hereafter) is located at  $l = 0$ . Shops in Area  $C$  supply continuum of different goods indexed by  $\theta$ . The range of  $\theta$  is  $[\underline{\theta}, \bar{\theta}]$ . The price of goods is  $p^c$  when consumers purchase a good in Area  $C$ .

Consumers purchase each type of good one unit at a time. The revenue of each shop in Area  $C$  is  $p^c N$ , where  $N$  is the number of consumers visiting Area  $C$ .  $N$  is determined by the consumer's shopping decisions, which are explained later. We assume that the costs of operating shops are different across types of goods, and are represented as a function of  $\theta$ ,  $c(\theta)$ .  $\theta$  is ordered according to the level of operating cost, such that a larger  $\theta$  means a greater cost ( $c'(\theta) > 0$ ). Shops in Area  $C$  may organize a shopping district association. The shopping district association collects the membership fee  $f$ . Thus type  $\theta$  shop's profit is  $\pi(\theta) = p^c N - c(\theta) - f$ . Since  $c(\theta)$  is an increasing function of  $\theta$ , the profit is a decreasing function of  $\theta$ . Let  $\theta^*$  be the break even level so that the type  $\theta^*$  shop's profit is just zero:

$$\pi(\theta^*) = p^c N - c(\theta^*) - f = 0. \quad (1)$$

$(\theta^* - \underline{\theta})$  shops whose type is in  $[\underline{\theta}, \theta^*]$  gain positive profit and continue their business in Area  $C$ .  $(\bar{\theta} - \theta^*)$  shops whose type is above  $\theta^*$  cannot get positive profit and are therefore shut down.

Consumers purchase  $(\bar{\theta} - \underline{\theta})$  types of goods one unit at a time and consume a composite good (numeraire). They can purchase the goods of  $[\underline{\theta}, \theta^*]$  either in Area  $C$  or in other areas. Shopping areas other than Area  $C$  are collectively treated as Area  $S$ . The goods of  $(\theta^*, \bar{\theta}]$  are not available in Area  $C$ , so consumers purchase them in Area  $S$ .

The utility of a consumer living at  $l \in [0, L]$  is

$$U(x^0, l) = x^0 + \int_{\underline{\theta}}^{\theta^*} [\delta(\theta, l) u^c(\theta^*, G) + (1 - \delta(\theta, l)) u^s] d\theta + \int_{\theta^*}^{\bar{\theta}} u^s d\theta. \quad (2)$$

$x^0$  is the consumption of the numeraire.  $\delta(\theta, l) = 1$  if consumers living at  $l$  purchase good  $\theta$  in area  $C$ .  $\delta(\theta, l) = 0$  otherwise.  $u^c(\theta^*, G)$  is the benefit derived from shopping in Area  $C$ . We assume that consumers enjoy the benefit every time they buy one unit of good in Area  $C$ .  $\theta^*$  represents the attractiveness of Area  $C$  as larger  $\theta^*$  implies that a greater variety of goods is available, while it also implies that there are fewer shutdown shops.  $G$  represents the public goods provided by

the shopping district association such as arcades, pavement, security, and event coordination. A shopping area with more public goods is more attractive to consumers. Thus,  $u^c(\theta^*, G)$  is an increasing function of  $\theta^*$  and  $G$ .

$u^s$  is the benefit consumers obtain when they purchase a good in Area  $S$ .  $u^s$  is assumed to be a constant.

Consumers' budget constraint is

$$x^0 + \int_{\underline{\theta}}^{\theta^*} [\delta(\theta, l)(p^c + t^c(l)) + (1 - \delta(\theta, l))(p^s + t^s)]d\theta + \int_{\theta^*}^{\bar{\theta}} [p^s + t^s]d\theta \leq y, \quad (3)$$

where  $t^c(l)$  is the transport cost for a consumer at  $l$  to visit Area  $C$ , which is an increasing function of  $l$ .  $p^s$  is the price of goods at Area  $S$ .  $t^s$  is the transport cost for shopping in Area  $S$ , which is assumed to be a constant. Transport cost is incurred every time consumers buy one unit of good.  $y$  is the income of each consumer.

We abstract the behavior of Shopping Area  $S$  in response to activities in Area  $C$ . Although suburban shopping malls also make efforts to attract more consumers, we do not consider this fact in our model because the focus of this study is on the role of shopping district associations.

Each consumer chooses  $\delta(\theta, l)$  to maximize the utility (2), subject to the budget constraint (3). Substituting (3) in (2), we can write the utility of consumers at  $l$  as follows:

$$U(l) = y + \int_{\underline{\theta}}^{\theta^*} [\delta(\theta, l)(u^c(\theta^*, G) - p^c - t^c(l)) + (1 - \delta(\theta, l))(u^s - p^s - t^s)]d\theta + \int_{\theta^*}^{\bar{\theta}} [u^s - p^s - t^s]d\theta. \quad (4)$$

Since consumers maximize  $U(l)$ ,  $\delta(\theta, l)$  is determined as follows:

$$\begin{aligned} \delta(\theta, G) = 1 &\iff u^c(\theta^*, G) - p^c - t^c(l) \geq u^s - p^s - t^s, \\ \delta(\theta, G) = 0 &\iff u^c(\theta^*, G) - p^c - t^c(l) < u^s - p^s - t^s. \end{aligned}$$

Consumers go shopping in Area  $C$  if and only if the net benefit of shopping in Area  $C$  is larger than that of shopping in Area  $S$ .

Let us denote by  $l^*(\in [0, L])$ , the market boundary at which consumers are indifferent to shopping either in Area  $C$  and shopping in Area  $S$ :

$$u^c(\theta^*, G) - p^c - t^c(l^*) = u^s - p^s - t^s. \quad (5)$$

Consumers in  $[0, l^*]$  purchase goods of  $[\underline{\theta}, \theta^*]$  in Area  $C$  and those of  $[\theta^*, \bar{\theta}]$  in Area  $S$ , while consumers in  $[l^*, L]$  do not visit Area  $C$  and purchase all the goods in Area  $S$ . The number of consumers visiting Area  $C$  is equal to the population in  $[0, l^*]$ ,  $N(l^*) = \int_0^{l^*} n(l)dl$ . Using this, equation (1) can be rewritten as

$$p^c N(l^*) - c(\theta^*) - f = 0. \quad (6)$$

(5) and (6) determine  $\theta^*$  and  $l^*$  as functions of  $f$  and  $G$ . Let us denote them as  $\theta^*(f, G)$  and  $l^*(f, G)$ .

The shopping district association is a kind of club in this model. In club theory (see, for example, Berglas [1]), only club members can enjoy the benefits of public goods provided by the club. In our model, nonmembers (consumers) enjoy the benefit of public goods provided by the association, and members (shops) indirectly receive the benefit through the increase of customers. This is the key difference between club theory and our model.

### 3 Equilibrium and Optimum

#### 3.1 Equilibrium Without the Shopping District Association

When shops in Area  $C$  do not form any association, membership fee  $f$  and the supply of public goods  $G$  are zero. So  $\theta^*(0, 0)$  and  $l^*(0, 0)$  are the values at the equilibrium without association. They are obtained by solving the following set of equations:

$$\begin{aligned} p^c N(l^*) - c(\theta^*) &= 0, \\ u^c(\theta^*, 0) - p^c - t^c(l^*) &= u^s - p^s - t^s. \end{aligned}$$

Without the association, each shop may supply some public goods voluntarily. However, since public goods such as arcades and events are indivisible and not provided by any individual shop, we can assume that the formation of an association is essential for the provision of public goods in the shopping area.

#### 3.2 Equilibrium With the Shopping District Association

Now, suppose shops in Area  $C$  organize an association to provide public goods. Assume that the shopping district association chooses  $f$  and  $G$  to maximize the



sum of the shops' profits in Area C. The association's maximization problem is

$$\max_{f, G} \int_{\underline{\theta}}^{\theta^*(f, G)} \pi(\theta) d\theta = \int_{\underline{\theta}}^{\theta^*(f, G)} (p^c N(l^*(f, G)) - c(\theta) - f) d\theta \quad (7.a)$$

$$\text{s.t. } G = f(\theta^*(f, G) - \underline{\theta}). \quad (7.b)$$

The constraint means that the amount of public goods provided by the association equals the total membership fees that the association collects from open shops. This is the budget constraint of the shopping district association.

(7.a) (7.b) is a constrained maximization problem with respect to  $f$  and  $G$ . We can restate it as an unconstrained maximization problem with respect to  $f$  by substituting the constraint (7.b) into the objective function (7.a). Accordingly,  $\theta^*(f, G)$ ,  $l^*(f, G)$ , and  $G$  are rewritten as  $\theta^*(f)$ ,  $l^*(f)$ , and  $G(f)$ . (7.a) and (7.b) can then be rewritten as

$$\max_f \int_{\underline{\theta}}^{\theta^*(f)} \pi(\theta) d\theta = \int_{\underline{\theta}}^{\theta^*(f)} (p^c N(l^*(f)) - c(\theta) - f) d\theta. \quad (8)$$

Before examining the first order condition of (8), we conduct comparative static analysis with respect to  $f$  from equations (5), (6), and (7.b):

$$\frac{d\theta^*}{df} = \frac{p^c n(l^*)(\theta^* - \underline{\theta}) \frac{\partial u^c}{\partial G} - t^{c'}(l^*)}{t^{c'}(l^*)c'(\theta^*) - p^c f n(l^*) \frac{\partial u^c}{\partial G} - p^c n(l^*) \frac{\partial u^c}{\partial \theta^*}}, \quad (9)$$

$$\frac{dl^*}{df} = \frac{(\theta^* - \underline{\theta})c'(\theta^*) \frac{\partial u^c}{\partial G} - f \frac{\partial u^c}{\partial G} - \frac{\partial u^c}{\partial \theta^*}}{t^{c'}(l^*)c'(\theta^*) - p^c f n(l^*) \frac{\partial u^c}{\partial G} - p^c n(l^*) \frac{\partial u^c}{\partial \theta^*}}. \quad (10)$$

The numerators can be either positive or negative. For a stable equilibrium, however, the denominator of these equations should be positive.<sup>1</sup>

The first-order condition of (8) is

$$p^c n(l^*(f^*)) \frac{dl^*}{df} = 1, \quad (11)$$

where  $f^*$  is the solution to the problem.<sup>2</sup> From (11),  $dl^*/df$  should be positive at  $f = f^*$ . In other words, the revenue of the shops should increase if the association raises  $f$  from  $f^*$ . When the association raises  $f$  marginally, the revenue of each

<sup>1</sup>The positiveness of the denominator follows from the well-known stability condition.

<sup>2</sup>We ignored the corner solution ( $\theta^*(f^*) = \underline{\theta}$ ) in which no shops are open in Area C.

shop increases by  $n(l^*(f^*))dl^*/df$ . Because there are  $(\theta^*(f^*) - \underline{\theta})$  open shops in Area C, the sum of the shops' revenue increases by  $(\theta^*(f^*) - \underline{\theta})n(l^*(f^*))dl^*/df$ , which is the marginal benefit of increasing  $f$ . At the same time, the sum of the shops' cost increases by  $(\theta^*(f^*) - \underline{\theta})$  because one dollar more in fees is imposed on each open shop. Equation (11) states that the marginal benefit of increasing  $f$  should be equalized to the marginal cost.

We can also interpret the first-order condition in a different way. Using equation (10), the first order condition (11) can be rewritten as

$$p^c(\theta^* - \underline{\theta})n(l^*)\frac{1}{t^{c'}(l^*)}\frac{\partial u^c}{\partial G} = 1. \quad (12)$$

When the association increases the public goods provision by one unit,  $l^*$ , the market boundary of Area C, spreads by  $\frac{1}{t^{c'}(l^*)}\frac{\partial u^c}{\partial G}$  units (see equation (5)). Then the number of consumers visiting Area C increases by  $n(l^*)\frac{1}{t^{c'}(l^*)}\frac{\partial u^c}{\partial G}$ . Since  $n(l^*)\frac{1}{t^{c'}(l^*)}\frac{\partial u^c}{\partial G}$  consumers visit  $(\theta^* - \underline{\theta})$  shops and buy one unit of goods (whose price is  $p^c$ ) at each shop, the increment of the total profits in Area C is  $p^c(\theta^* - \underline{\theta})n(l^*)\frac{1}{t^{c'}(l^*)}\frac{\partial u^c}{\partial G}$  while the cost to provide one more unit of public goods is unity. The marginal benefit of public goods is equal to their marginal cost when equation (12) holds.

**PROPOSITION 1** *When the shopping district maximizes its total profit, the number of open shops in Area C is also maximized.*

**(proof)** From (9) and (12),  $d\theta^*/df = 0$  at the equilibrium with association. The number of open shops  $(\theta^*(f) - \underline{\theta})$  is maximized at the equilibrium, where second order conditions are supposed to be satisfied. **Q.E.D.**

Note that it is a maximization under the constraint that the provision of public goods is afforded by membership fees. More shops can be open without this constraint.

We can also compare the welfare of consumers and firms at the equilibrium with and without a shopping district association.

**PROPOSITION 2** *No shop in Area C decreases its profit and no consumer decreases her welfare thorough the establishment of a shopping district association.*

**(proof)** Proposition 1 tells us  $\theta^*(0) < \theta^*(f^*)$ . Thus,  $p^cN(l^*(0)) < p^cN(l^*(f^*)) - f^*$  follows, because  $p^cN(l^*(0)) = c(\theta^*(0)) < c(\theta^*(f^*)) = p^cN(l^*(f^*)) - f^*$ . Without an association, shops whose type is in  $[\underline{\theta}, \theta^*(0)]$  are open. Their profits increase from  $p^cN(l^*(0)) - c(\underline{\theta})$  to  $p^cN(l^*(f^*)) - c(\underline{\theta}) - f^*$  when an association is established.

Shops in  $(\theta^*(0), \theta^*(f^*))$  shut down when there is no association and remain open when there is. Their profits increase from zero to  $p^c N(l^*(f^*)) - c(\theta) - f^*$ . Shops in  $(\theta^*(f^*), \bar{\theta}]$  remain closed after the establishment of an association. No shop in Area  $C$  experiences profit loss.

The utility of consumers in  $[0, l^*(0)]$  increases from

$$y + \int_{\underline{\theta}}^{\theta^*(0)} (u^c(\theta^*(0), 0) - p^c - t^c(l))d\theta + \int_{\theta^*(0)}^{\bar{\theta}} (u^s - p^s - t^s)d\theta$$

to

$$y + \int_{\underline{\theta}}^{\theta^*(f^*)} (u^c(\theta^*(f^*), G(f^*)) - p^c - t^c(l))d\theta + \int_{\theta^*(f^*)}^{\bar{\theta}} (u^s - p^s - t^s)d\theta.$$

The utility of consumers in  $(l^*(0), l^*(f^*))$  increases from

$$y + \int_{\underline{\theta}}^{\bar{\theta}} (u^s - p^s - t^s)d\theta$$

to the above level. The utility of consumers in  $(l^*(f^*), L]$  does not change. No consumer experiences welfare loss. **Q.E.D.**

Proposition 2 shows that the formation of the association makes all the consumers and all the shops in Area  $C$  better off (or, at least, not worse off). Note that this is not a Pareto improvement because the profits of shops in Area  $S$  decrease.

### 3.3 First-Best Allocation

In this paper, the first-best optimum is defined as the allocation that maximizes the social surplus, which is the sum of consumer surplus, shops' profits, and the association's fiscal surplus, i.e.,  $SW = \Pi + CS + \Pi^s + A$ .  $\Pi$  is the profits of shops in Area  $C$ :

$$\Pi = \int_{\underline{\theta}}^{\theta^*} (p^c N(l^*) - c(\theta) - f)d\theta.$$

$CS$  is the consumer surplus:

$$\begin{aligned} CS = & \int_0^{l^*} [y + (\theta^* - \underline{\theta})(u^c(\theta^*, G) - p^c - t^c(l)) + (\bar{\theta} - \theta^*)(u^s - p^s - t^s)]n(l)dl \\ & + \int_{l^*}^L [y + (\bar{\theta} - \underline{\theta})(u^s - p^s - t^s)]n(l)dl, \end{aligned}$$

where the first term of the RHS is the surplus of consumers in  $[0, l^*]$  and the second term is the surplus of consumers in  $[l^*, L]$ .  $\Pi^S$  is the profits of shops in Area  $S$ :

$$\Pi^S = (\bar{\theta} - \theta^*)p^S N(L) + (\theta^* - \underline{\theta})p^S (N(L) - N(l^*)) - c^S(\bar{\theta} - \underline{\theta}).$$

The first term of the RHS means the sum of revenue by supplying goods of  $[\theta^*, \bar{\theta}]$  to all consumers. The second term means the sum of revenue by supplying goods of  $[\underline{\theta}, \theta^*]$  to the consumers in  $[l^*, L]$ . The last term is their operating cost. Shops in Area  $S$  provide  $(\bar{\theta} - \underline{\theta})$  types of goods.  $c^S$  is the operating cost to provide one type of good and is assumed to be a constant.  $A$  is the fiscal surplus of the shopping district association,  $A = (\theta^* - \underline{\theta})f - G$ .

At the first best, the social welfare is maximized with respect to  $\theta^*$ ,  $l^*$ , and  $G$ . The first order conditions are

$$-c(\theta^*) + \left( (\theta^* - \underline{\theta}) \frac{\partial u^c}{\partial \theta^*} + u^c(\theta^*, G) - (u^S - t^S) \right) N(l^*) - \int_0^{l^*} t^c(l)n(l)dl = 0, \quad (13)$$

$$u^c(\theta^*, G) - t^c(l^*) = u^S - t^S, \quad (14)$$

$$\frac{\partial u^c}{\partial G}(\theta^* - \underline{\theta})N(l^*) = 1. \quad (15)$$

The LHS of equation (13) represents the marginal (net) benefit of an increase in  $\theta^*$ . The first term of the LHS of (13) is the cost of one more open shop, i.e. the operating cost of the marginal shop. The second term is the increment of consumer surplus by a marginal increase in  $\theta^*$ . The third term is the increment of consumers' transport cost. This is an opportunity cost for consumers who shift the shopping destination from Area  $S$  to Area  $C$ . Note that the increment of shops' revenue and the increment of consumers' expenditure cancel each other out in (13).

Equation (14) indicates that the marginal benefit and cost of extending Area  $C$ 's market are equalized. If Area  $C$ 's market is extended by one unit,  $n(l^*)$  more consumers will shop in Area  $C$ . Their benefit is  $(u^c(\theta^*, G) - t^c(l^*))n(l^*)$ . On the other hand, they stop shopping in Area  $S$ . Consumer's benefit decreases by  $(u^S - t^S)n(l^*)$ . The marginal (net) benefit is zero at the first best.

Equation (15) expresses the trade-off inherent in supplying public goods. The LHS is the marginal increase in the benefit to consumers that comes from increasing the provision of public goods. The RHS is the marginal cost of providing public goods. This equation is equivalent to Samuelson's condition for the optimal provision of public goods.

What are the differences between the first-best allocation and the equilibrium? Let us investigate how to decentralize the first-best allocation through taxes and subsidies. First, compare optimal condition (13) with equilibrium condition (6). Equation (13) is thus rewritten as

$$p^c N(l^*) - c(\theta^*) + \left[ \int_0^{l^*} \left( (\theta^* - \underline{\theta}) \frac{\partial u^c}{\partial \theta^*} + (u^c(\theta^*, G) - p^c - t^c(l)) - (u^s - p^s - t^s) \right) n(l) dl - p^s N(l^*) \right] = 0 \quad (16)$$

If the membership fee ( $-f$ ) is set as equal to the last term (in the square brackets) of the LHS of (16), the first best  $\theta^*$  can be achieved. The integral in the bracket is the marginal benefit to consumers of there being one more open shop in Area  $C$ . The last term in the bracket is the decrease in revenue of the shops in Area  $S$  that is caused by a shift of consumers to Area  $C$  in response to larger  $\theta^*$ . At the equilibrium, the shops in Area  $C$  decide if they will stay open without considering the positive externality to consumers (and negative externality to shops in Area  $S$ ). If the fee is set as above, these externalities are internalized. Since the integral term in the bracket is positive and the last term is negative, the sign of the sum is ambiguous. If the sum is positive, the membership fee becomes negative, which implies a subsidy to the shops in Area  $C$ .

Second, compare optimal condition (14) and equilibrium condition (5). If consumers receive a subsidy of  $(p^c - p^s)$  for every unit of goods purchased in Area  $C$ , the first-best market boundary  $l^*$  can be achieved.<sup>3</sup>

Last, compare equation (15) to equilibrium condition (12). The LHS of (12) is the marginal increase of total profits in Area  $C$  by additional public goods supply, as mentioned above. The LHS of the first-best condition (15) is the benefit to consumers that comes from the additional public goods. The subsidy that achieves the first-best public goods supply is equal to the difference between the marginal benefit of consumers and the marginal profits of shops in Area  $C$ ,  $Q = \frac{\partial u^c}{\partial G}(\theta^* - \underline{\theta}) \left( N(l^*) - \frac{p^c n(l^*)}{t^c(l^*)} \right)$ .<sup>4</sup>

In summary, a subsidy to (or a tax on) individual shops in Area  $C$  internalizes the externality caused by  $\theta^*$ , and a subsidy to (or a tax on) the shopping district association eliminates the welfare loss from inefficient provision of public goods.

<sup>3</sup>Subsidy (or tax) to consumers is necessary only if the price of goods is different between Area  $C$  and Area  $S$ .

<sup>4</sup>The sign of  $Q$  is ambiguous but is likely to be positive because it is natural that the direct benefit received by consumers exceeds the indirect benefit received by shops.

In reality, however, the first-best allocation is hardly achieved, because the information about, for example, how consumers benefit from the provision of public goods and the number of open shops is not available. Moreover, the subsidy to individual shops in Area  $C$  is not practical. We introduce the alternative, second-best policies in the next section.

## 4 Effect of Government Intervention

We will examine three types of government intervention policies. The first type is the optimal fee policy, whereby the government controls the membership fee  $f$  and the public goods provision  $G$ . This policy will reduce the welfare loss from inefficient provision of public goods to some extent because the government chooses  $f$  and  $G$  to maximize social welfare. The second type is the matching subsidy, whereby the government subsidizes a part of the association's expenditure. This policy also reduces the welfare loss from inefficient provision of public goods. Under this policy, we can expect that the public good provision increases if it is too small at the equilibrium.<sup>5</sup> The third type of government intervention is the transport-cost subsidy, whereby the government subsidizes a portion of transportation costs of the consumers when they shop in Area  $C$ . This subsidy will let more consumers visit Area  $C$  and thus increase the sales of shops there. The association will then be able to keep more shops open than those at the equilibrium.

### 4.1 Optimal Membership Fee: Optimal Tax from Shops

First, we examine the case in which the government controls the membership fee of the association. Note that  $\theta^*$ ,  $l^*$ , and  $G$  are determined as functions of  $f$  according to equations (5), (6), and (7.b). Thus, social welfare is also represented as a function of  $f$ . The government decides  $f$  to maximize social welfare,  $SW(f)$ . In this case, the government, not the association, collects membership fees directly from shops and provides public goods. The membership fee is therefore equivalent to a tax.

The government's maximization problem is therefore

$$\max_f SW(f) = \Pi(f) + CS(f) + \Pi^s(f),$$

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<sup>5</sup>It is possible that the public good provision is too large in the equilibrium.

where

$$\begin{aligned}
\Pi(f) &= \int_{\underline{\theta}}^{\theta^*(f)} (p^c N(l^*(f)) - c(\theta) - f) d\theta, \\
CS(f) &= \int_0^{l^*(f)} [y + (\theta^*(f) - \underline{\theta})(u^c(\theta^*(f), G(f)) - p^c - t^c(l)) + (\bar{\theta} - \theta^*(f))(u^s - p^s - t^s)] n(l) dl \\
&\quad + \int_{l^*(f)}^L [y + (\bar{\theta} - \underline{\theta})(u^s - p^s - t^s)] n(l) dl, \\
\Pi^s(f) &= (\bar{\theta} - \theta^*(f)) p^s N(L) + (\theta^*(f) - \underline{\theta}) p^s (N(L) - N(l^*(f))) - c^s(\bar{\theta} - \underline{\theta})
\end{aligned}$$

The first-order condition is

$$\begin{aligned}
&(\theta^* - \underline{\theta}) \left( p^c n(l^*) \frac{dl^*}{df} - 1 \right) \\
&+ \int_0^{l^*} \left[ \frac{d\theta^*}{df} ((u^c(\theta^*, G) - p^c - t^c(l)) - (u^s - p^s - t^s)) + (\theta^* - \underline{\theta}) \left( \frac{\partial u^c}{\partial \theta^*} \frac{d\theta^*}{df} + \frac{\partial u^c}{\partial G} \frac{dG}{df} \right) \right] n(l) dl \\
&- p^s \frac{d\theta^*}{df} N(l^*) - (\theta^* - \underline{\theta}) p^s n(l^*) \frac{dl^*}{df} = 0. \tag{17}
\end{aligned}$$

The first term of the LHS is the variation of Area C's profit, which is positive if  $f < f^*$  and negative if  $f > f^*$ .

The second line of (17) shows the variation of the benefit to consumers in  $[0, l^*(f)]$ . The first term in the square brackets represents the net benefit of shifting the shopping destination for  $\theta^*$ , which is positive if  $f < f^*$  and negative if  $f > f^*$  depending on the sign of  $\left(\frac{d\theta^*}{df}\right)$ . The second term in the square brackets is the effect of  $f$  on the benefit to consumers of purchasing  $[\underline{\theta}, \theta^*]$  goods in Area C through the change of  $\theta^*$  and  $G$ .  $\left(\frac{\partial u^c}{\partial \theta^*} \frac{d\theta^*}{df}\right)$  is positive if  $f < f^*$  and negative if  $f > f^*$ .  $\left(\frac{\partial u^c}{\partial G} \frac{dG}{df}\right)$  is usually positive.<sup>6</sup>

The two terms in the third line of (17) are the variation of Area S's profit. The third term is negative if  $f < f^*$  and positive if  $f > f^*$ . The last term is usually negative.<sup>7</sup>

The LHS of (17) is  $dS W/df$ . At the equilibrium value  $f = f^*$ ,

$$\left. \frac{dS W}{df} \right|_{f=f^*} = (\theta^* - \underline{\theta}) \frac{\partial u^c}{\partial G} \frac{dG}{df} N(l^*) - (\theta^* - \underline{\theta}) p^s n(l^*) \frac{dl^*}{df}.$$

<sup>6</sup>  $\frac{dG}{df} < 0$  is possible (i.e. the public goods may decrease) in large  $f$ .

<sup>7</sup>  $\frac{dl^*}{df} < 0$  is possible (i.e. the number of consumers visiting Area C may decrease) in large  $f$ .

The first term is the variation of  $CS$ , which is positive. The second term is a variation of  $\Pi^s$ , which is negative. When the membership fee  $f$  is marginally raised from  $f^*$ , the consumer surplus increases because there are more public goods provided. On the other hand, the profit in Area  $S$  decreases because  $dl^*/df > 0$  at  $f = f^*$  from (11). Under this policy, the number of open shops,  $\theta^*$ , and the total profits in Area  $C$ ,  $\Pi$ , must decrease from the equilibrium values because  $\theta^*(f)$  and  $\Pi(f)$  are maximized at  $f^*$ .

It is unclear whether the optimal fee is larger or smaller than the equilibrium value  $f^*$  (see Figure 1). If we let  $f^o$  be the optimal fee and  $f^{CS}$  be the value maximizing the consumer surplus  $CS(f)$ ,<sup>8</sup>  $\Pi(f)$  and  $CS(f)$  are maximized at  $f^*$  and  $f^{CS}$ , respectively. The profits of shops in Area  $S$ ,  $\Pi^s(f)$ , are minimized at a value in  $(f^*, f^{CS})$ .<sup>9</sup>  $f^o$  maximizes the sum of these three functions. Figure 1 shows three possible cases of  $f^o$  value: (i)  $f^o \leq f^*$ , (ii)  $f^* < f^o < f^{CS}$ , and (iii)  $f^{CS} \leq f^o$ . If the variation of  $\Pi^s(f)$  is small enough, (ii)  $f^* < f^o < f^{CS}$  should be the result.

In case (i), the optimal fee policy decreases  $\Pi$  and  $CS$ , while  $\Pi^s$  increases from the equilibrium. In case (ii),  $\Pi$  decreases but  $CS$  increases. Whether  $\Pi^s$  increases or decreases is unclear. In case (iii), the optimal fee policy decreases  $\Pi$  while it increases either  $CS$  or  $\Pi^s$  (or both).

## 4.2 Matching Subsidy to the Association

Let us now consider another second-best policy: matching subsidy to the association. In this scenario, the government bears a part of the association's expenditure. The policy promotes the provision of public goods by the shopping district association if it is too small in the equilibrium. In reality, national or local government in Japan subsidizes 1/2 to 2/3 of the cost for some projects planned by the association, such as constructing an arcade along the street, preparing an area security system, and transforming a shutdown shop into a shop that sells local products.

Let the rate of subsidy be  $\alpha (\in [0, 1])$ . (7.b) then becomes

$$(1 - \alpha)G = f(\theta^* - \underline{\theta}). \quad (18)$$

The association decides  $f$  and  $G$  to maximize (7.a) under constraint (18).  $\alpha$  is determined by the government.

<sup>8</sup>We can show  $f^* < f^{CS}$ .  $\frac{dCS}{df} = (\theta^* - \underline{\theta})^2 \frac{\partial u^c}{\partial G} N(l^*) > 0$  at  $f = f^*$ .

<sup>9</sup>We can show it.  $\frac{d\Pi^s}{df} = -(\theta^* - \underline{\theta}) \frac{p^s}{p^c} < 0$  at  $f = f^*$  and  $\frac{d\Pi^s}{df} = -p^s \frac{d\theta^*}{df} N(l^*) > 0$  at  $f = f^{CS}$ .



The association's first order condition is similar to equation (11),

$$p^c n(l^*(f, \alpha)) \frac{\partial l^*}{\partial f} = 1. \quad (19)$$

We can denote the solution of the above equation by  $f(\alpha)$ , which is a function of  $\alpha$ .

Now we can examine the effect of  $\alpha$  on  $\theta^*$  and other variables:

$$\begin{aligned} \frac{d\theta^*}{d\alpha} &= \frac{\partial \theta^*}{\partial f} \frac{df}{d\alpha} + \frac{\partial \theta^*}{\partial \alpha} \\ &= \frac{\partial \theta^*}{\partial \alpha} \\ &= \frac{p^c n(l^*) \frac{\partial u^c}{\partial G} G}{(1 - \alpha) t^{c'}(l^*) c'(\theta^*) - p^c n(l^*) f \frac{\partial u^c}{\partial G} - (1 - \alpha) p^c n(l^*) \frac{\partial u^c}{\partial \theta^*}}. \end{aligned}$$

$\frac{d\theta^*}{d\alpha} > 0$  since the denominator is positive if we assume the equilibrium is stable as shown in section 3.2. Thus, the number of open shops in Area  $C$  increases when the government introduces the matching subsidy policy.

The total profit in Area  $C$  is

$$\Pi(f(\alpha), \alpha) = \int_{\underline{\theta}}^{\theta^*(f(\alpha), \alpha)} (p^c N(l^*(f(\alpha), \alpha) - c(\theta) - f(\alpha)) d\theta.$$

Differentiating  $\Pi(\cdot)$  with respect to  $\alpha$  and using (19), we have

$$\begin{aligned} \frac{d\Pi}{d\alpha} &= (\theta^* - \underline{\theta}) \left( p^c n(l^*) \left( \frac{\partial l^*}{\partial f} \frac{df}{d\alpha} + \frac{\partial l^*}{\partial \alpha} \right) - \frac{df}{d\alpha} \right) \\ &= (\theta^* - \underline{\theta}) p^c n(l^*) \frac{\partial l^*}{\partial \alpha} \\ &= (\theta^* - \underline{\theta}) p^c n(l^*) \frac{\frac{\partial u^c}{\partial G} c'(\theta^*) G}{(1 - \alpha) t^{c'}(l^*) c'(\theta^*) - p^c n(l^*) f \frac{\partial u^c}{\partial G} - (1 - \alpha) p^c n(l^*) \frac{\partial u^c}{\partial \theta^*}}, \end{aligned}$$

which is positive. The total profit in Area  $C$  increases when the government introduces the matching subsidy.

The signs of  $(df/d\alpha)$ ,  $(dl^*/d\alpha)$ , and  $(dG/d\alpha)$  are ambiguous. If  $df/d\alpha > 0$ ,  $(dl^*/d\alpha)$  and  $(dG/d\alpha)$  are known to be positive. However,  $df/d\alpha < 0$  must hold for some  $\alpha$  because  $\lim_{\alpha \rightarrow 0} f = f^* > 0$  and  $\lim_{\alpha \rightarrow 1} f = 0$ . When  $df/d\alpha < 0$  and  $dG/d\alpha > 0$ ,  $dl^*/d\alpha > 0$  because both the number of open shops,  $\theta^*$ , and

the provision of public goods,  $G$ , increases. When  $df/d\alpha < 0$  and  $dG/d\alpha < 0$ , both  $dl^*/d\alpha > 0$  and  $dl^*/d\alpha < 0$  are possible. It may happen that the number of consumers visiting Area  $C$ ,  $l^*$ , and the public goods supply,  $G$ , both decrease when the government introduces the matching subsidy. In this case, the subsidy benefits shops both in Area  $C$  and in Area  $S$  at the expense of consumers and taxpayers.

The sign of  $dCS/d\alpha$  is undetermined.  $d\Pi^s/d\alpha < 0$  when  $df/d\alpha > 0$ .

The government chooses  $\alpha$  to maximize social welfare taking into account the association's response,  $f(\alpha)$ . The government's maximization problem is

$$\max_{\alpha} SW(f(\alpha), \alpha) = \Pi(f(\alpha), \alpha) + CS(f(\alpha), \alpha) + \Pi^s(f(\alpha), \alpha) - \alpha G(f(\alpha), \alpha),$$

where the last term is the government's expenditure.

The first-order condition with respect to  $\alpha$  is

$$\begin{aligned} & (\theta^* - \underline{\theta}) \left( p^c n(l^*) \frac{dl^*}{d\alpha} - \frac{df}{d\alpha} \right) \\ & + \int_0^{l^*} \left[ \frac{d\theta^*}{d\alpha} ((u^c(\theta^*, G) - p^c - t^c(l)) - (u^s - p^s - t^s)) + (\theta^* - \underline{\theta}) \left( \frac{\partial u^c}{\partial \theta^*} \frac{d\theta^*}{d\alpha} + \frac{\partial u^c}{\partial G} \frac{dG}{d\alpha} \right) \right] n(l) dl \\ & - p^s \left( \frac{d\theta^*}{d\alpha} N(l^*) + \frac{dl^*}{d\alpha} (\theta^* - \underline{\theta}) n(l^*) \right) - \alpha \frac{dG}{d\alpha} - G = 0. \end{aligned}$$

What each term indicates is almost the same as the first-order condition of optimal fee policy (17). The last two terms are the variation in the government's expenditure.

### 4.3 Transport-Cost Subsidy to Consumers

Let us consider an alternative form of public policy, a transport-cost subsidy. The government subsidizes a portion of consumers' transportation costs when they go shopping in the central shopping area. More consumers will come to Area  $C$  if this policy is in effect, and therefore the profits in Area  $C$  will increase, which in turn enables the association to increase the provision of public goods and the number of open shops in the area, and attract even more consumers. The policy can be implemented through reduced public transit fares or the creation of low-cost public parking.

We assume that the government pays  $r$  for each visit to Area  $C$ .<sup>10 11</sup> Equation

<sup>10</sup>In our model,  $r$  is paid for each unit of goods consumers purchase in Area  $C$ .

<sup>11</sup>A transport cost subsidy proportional to the distance between the consumer and the central

(5) thus becomes

$$u(\theta^*, G) - p^c - t^c(l^*) + r = u^s - p^s - t^s. \quad (20)$$

Equations (20), (6), and (7.b) determine  $\theta^*$ ,  $l^*$ , and  $G$  as functions of  $f$  and  $r$ . Given the subsidy rate  $r$ , the association chooses  $f$  to maximize the total profit. The first order condition does not change:

$$p^c n(l^*(f, r)) \frac{\partial l^*}{\partial f} = 1.$$

The optimal value of  $f$  depends on the level of  $r$ ; we denote it as  $f(r)$ .

We can see  $d\theta^*/dr > 0$  and  $d\Pi/dr > 0$  in the same way as we determined the effect of the matching subsidy policy. In general, we cannot determine the signs of  $(df/dr)$ ,  $(dl^*/dr)$ , and  $(dG/dr)$ , but  $df/dr > 0$ ,  $dl^*/dr > 0$  and  $dG/dr > 0$  hold if we specify the functional forms as in Section 5. If a transport-cost subsidy is introduced, more consumers will visit Area  $C$  and the profits in Area  $C$  will thus increase. More shops can open in Area  $C$  and the association will increase the provision of public goods.

Suppose that the government chooses  $r$  to maximize social welfare. The government's maximization problem is

$$\begin{aligned} \max_r \quad & SW(f(r), r) = \Pi(f(r), r) + CS(f(r), r) + \Pi^s(f(r), r) \\ & - r(\theta^*(f(r), r) - \underline{\theta})N(l^*(f(r), r)). \end{aligned}$$

The last term is the government's expenditure (the subsidy that consumers receive). The first order condition for the government is

$$\begin{aligned} & (\theta^* - \underline{\theta}) \left( p^c n(l^*) \frac{dl^*}{dr} - \frac{df}{dr} \right) \\ & + \int_0^{l^*} \left[ \frac{d\theta^*}{dr} ((u^c(\theta^*, G) - p^c - t^c(l)) - (u^s - p^s - t^s)) + (\theta^* - \underline{\theta}) \left( \frac{\partial u^c}{\partial \theta^*} \frac{d\theta^*}{dr} + \frac{\partial u^c}{\partial G} \frac{dG}{dr} \right) \right] n(l) dl \\ & - p^s \left( \frac{d\theta^*}{dr} N(l^*) + \frac{dl^*}{dr} (\theta^* - \underline{\theta}) n(l^*) \right) - r(\theta^* - \underline{\theta}) n(l^*) \frac{dl^*}{dr} = 0. \end{aligned}$$

What each term means is virtually the same as in the case of an optimal-fee policy. The last term is the increment of public expenditure.<sup>12</sup>

shopping area is also possible. If the government lowers the fares of public transportation, consumers who live far from the city center will get more benefit from this policy.

<sup>12</sup>We also considered the transport cost subsidy by the shopping district association. But we found that such subsidies do not increase the total profits of the central shopping area so the association does not have an incentive for such a policy in our model.

## 5 Evaluation of Alternative Policies

We introduced three types of second-best policies in Section 4. This section evaluates these alternative policies through examining their effects on social welfare.

We conducted a numerical simulation for specific functional forms and parameter values. Functional forms are specified as follows:

$$\begin{aligned} n(l) &= 1 \quad \text{for all } l \in [0, L], \\ c(\theta) &= c\theta \quad (c > 0), \\ t^c(l) &= t^c l \quad (t^c > 0), \\ u^c(\theta^*, G) &= a\theta^* + b\sqrt{G} + d \quad (a, b, d > 0). \end{aligned}$$

Under these functional forms, equations (5), (6), and (7.b) are rewritten as:

$$a\theta^* + b\sqrt{G} + d - p^c - t^c l^* = u^s - p^s - t^s, \quad (21)$$

$$p^c l^* - c\theta^* - f = 0, \quad (22)$$

$$G = f(\theta^* - \underline{\theta}). \quad (23)$$

Parameters for the numerical simulation are chosen as follows:

$$\begin{aligned} L = 3, \quad \underline{\theta} = 0.1, \quad a = 0.5, \quad b = 0.5, \quad c = 4, \quad d = 3, \quad c^s = 2, \\ \bar{\theta} = 1, \quad t^c = 1.5, \quad t^s = 2, \quad u^s = 3, \quad y = 3, \quad p^c = 1, \quad p^s = 1. \end{aligned}$$

Table 1 shows the calculation results for six cases: the equilibrium without association, the equilibrium with association, the first best allocation, and the three second-best policies.

$\Pi$  is the total profit of shops in Area  $C$ .  $CS$  is the consumer's surplus.  $\Pi^s$  is the total profit in Area  $S$ .  $SW$  is social welfare which is  $\Pi + CS + \Pi^s -$  (public expenditure).

$\gamma$  in the last row of the table, which is an index of welfare improvement, is defined as

$$\gamma = \frac{(SW \text{ in each row}) - (SW \text{ at the Equilibrium without Association})}{(SW \text{ at the First Best}) - (SW \text{ at the Equilibrium without Association})}.$$

First, compare the equilibrium with and without an association. The creation of the shopping district association increases the number of open shops in the area,  $\theta^*$ , and consumers visiting the area,  $l^*$ . It also increases total profits in the area,  $\Pi$ , the consumer surplus,  $CS$ , and the social welfare,  $SW$ . These results show

that a shopping district association has a positive effect on the revitalization of the central shopping area and also improves social welfare.

At the equilibrium, the association faces a trade-off between the provision of public goods and the number of open shops in the area. To increase the provision of public goods from the equilibrium level, the association should raise  $f$  to finance larger expenditures. Raising  $f$  should, however, induce the closure of more shops in the area.

Compare the equilibrium to the first-best allocation case. In the first-best allocation case, both the number of open shops and the provision of public goods increase from the equilibrium values. There is no trade-off between them in the optimum because the financial constraint that the revenue from fees should cover the cost of the provision of public goods does not exist. Two types of subsidies are required to achieve the first-best allocation. One is the subsidy to each open shop, which internalizes the externality caused by the number of open shops. The value of the subsidy is 0.6544 in the numerical analysis. The other is the subsidy to the association, which removes the distortion caused by the inefficient provision of public goods. This subsidy is  $Q = 1.2050$  in this numerical example.

Now look at the second-best policies. Compare the optimal fee policy to the equilibrium. Under this policy, the fee is higher than that at the equilibrium. The optimal fee  $f^o$  is larger than the equilibrium fee  $f^*$  in this example.  $f^* < f^o < f^{CS}$  (case (ii) in Figure 1) holds true here.<sup>13</sup> Consequently the open shops are fewer under the optimal fee than at the equilibrium. An optimal fee policy improves social welfare mainly through an increase in the provision of public goods.

The matching subsidy policy increases both the number of open shops,  $\theta^*$ , and the total profit in Area C,  $\Pi$ , compared with the equilibrium as shown in subsection 4.2. This result is in contrast to the optimal fee policy, in which the number of open shops decreases from the equilibrium value. The membership fee,  $f$ , the provision of public goods,  $G$ , and the number of consumers visiting area C,  $l^*$ , are higher than the equilibrium values in this numerical example. The table shows that the matching subsidy policy attains higher social welfare than the optimal fee policy. The combination of the association and the government (the matching subsidy) achieves a better result than either the association alone (the equilibrium) or the government alone (the optimal fee).

Under the transport-cost subsidy policy, the number of open shops,  $\theta^*$ , and the total profit in Area C,  $\Pi$ , increase from the equilibrium. The membership fee,  $f$ , the provision of public goods,  $G$ , and the number of consumers visiting

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<sup>13</sup>The fee maximizing the consumer's surplus,  $f^{CS}$ , is 0.0329 in the example.

Area  $C$ ,  $l^*$ , also increase under the specified functional forms. These facts verify the analytical result in subsection 4.3. In the numerical analysis,  $\theta^*$ ,  $l^*$ , and  $\Pi$  in this case are much higher than the values at equilibrium and under the other two second-best policies, while  $f$  and  $G$  are only slightly greater than the equilibrium values. The transport-cost subsidy increases the number of consumers visiting Area  $C$  so effectively that the association does not need to increase the provision of public goods. In this sense, the provision of public goods and a transport-cost subsidy are substitutable policy instruments for attracting more consumers. A reduced level of public goods provision translates into fewer membership fees and more open shops, which further increases the number of consumers visiting Area  $C$ . Social welfare is also the highest among the three second-best policies, although the government's expenditure is very large under this policy.

Social welfares attained by the second-best policies are much less than in the first-best case. One main difference between the two scenarios is that the shops do not pay any fees and receive a subsidy in the first-best case, while the second-best policies impose a membership fee on the shops.

## 6 Conclusion

Voluntary organizations such as BIDs and shopping district associations are believed to provide a solution to the decline of traditional central shopping areas. We have shown the effects and limitations of shopping district associations.

Shopping district associations provide various public goods that no individual shop on its own could provide. The number of shutdown shops in the area is decreased by the formation of an association. We show that the association improves the profitability of shops in the area and the welfare of consumers. Shopping district associations therefore play a definite role in revitalizing central shopping areas and keeping city centers from deteriorating.

On the other hand, however, there are limitations to what the associations can do. Equilibrium with an association is inefficient because the association is not concerned with the positive externalities received by consumers. The government can internalize the externalities and improve social welfare using second-best policies. We examined optimal membership fee policy, matching subsidy policy, and the transport-cost subsidy policy, and evaluated them using numerical analysis.

One possible extension is to introduce land market, which will enable us to deal with the effect of other policies such as tax increment financing. Another outstanding problem is that we did not analyze the process of association forma-

tion. Our model assumes that all shops join the association; i.e. participation in the association is compulsory. If participation is actually voluntary, however, individual shops may not have enough of an incentive to join the association. They are better off not paying the membership fees and simply enjoying the public goods provided by the association. However, if all shops in the area did this, the association would not be able to sustain itself. What kind of incentive might entice the shops to pay the association's membership fee and to contribute to the provision of public goods? And how are voluntary associations sustainable? Answering these questions is our future works.

Table 1: Results of Numerical Analysis

	$f$	$\theta^*$	$l^*$	$G$	$\Pi$	$CS$	$\Pi^s$	$SW$	$\gamma$	public expenditure
Equilibrium Without Association	0	0.3636	1.4546	0	0.1390	9.4183	0.5165	10.0739	0	0
Equilibrium with Association	0.0074	0.3656	1.4700	0.0020	0.1411	9.4305	0.5095	10.0812	0.1525	0
First Best	-0.6544	0.5586	1.5799	0.0328	0.1205	9.8585	0.1755	10.1217	1	0.3001
Optimal Fee	0.0256	0.3641	1.4821	0.0068	0.1395	9.4352	0.5085	10.0832	0.1955	0
Matching Subsidy ( $\alpha=0.59$ )	0.0182	0.3689	1.4936	0.0119	0.1446	9.4493	0.4984	10.0852	0.2381	0.0070
Transport Cost Subsidy ( $r=0.230$ )	0.0085	0.4078	1.6397	0.0026	0.1895	9.6206	0.3953	10.0893	0.3222	0.1161



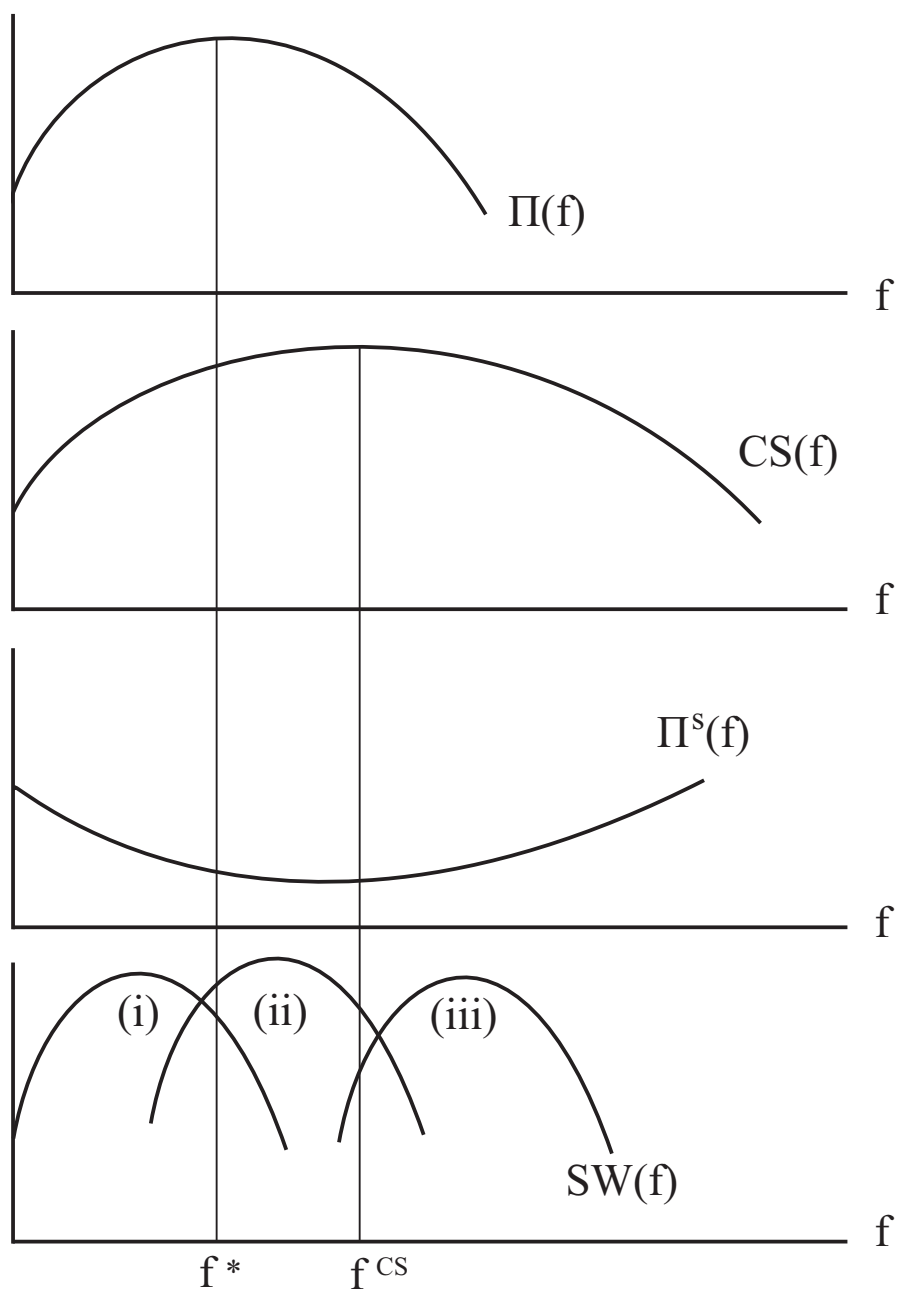


Figure 1: Three cases of welfare maximizing value  $p^o$

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